

Maple packages for the analysis of linear systems of partial difference equations and applications

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This talk reviews work by the speaker and coauthors on Maple packages for the analysis of linear systems of partial difference equations and related applications [6, 11, 7, 8, 2].

The Maple package `LDA` [6, 8] has been developed by V. P. Gerdt and the speaker since 2005. It implements algorithms for computing involutive bases [4] for linear difference ideals and, more generally, for submodules of finitely generated free left modules over (not necessarily commutative) rings of difference operators with coefficients in a difference field of characteristic zero.

A first application is the symbolic generation of finite difference schemes for linear PDEs [6, 8]. An elementary integration method is applied to the integral of the corresponding conservation equation, and in the resulting difference equations the partial derivatives of the unknown function are eliminated in favor of the unknown function itself. Following this idea, standard difference schemes for the Laplace, heat, wave, and advection equations and several difference schemes for the Burgers' and Falkowich-Kármán equations have already been generated symbolically in [5]. A second application is the reduction of Feynman integrals, which satisfy certain linear recurrence relations. Another application is a formal computational check of consistency of finite difference approximations of linear systems of PDEs [7].

The Maple package `OreModules` [2] has been developed by F. Chyzak, A. Quadrat and the speaker since 2003. It implements methods of module theory and homological algebra for Ore algebras. Modules over rings of difference operators form a special case to which these methods can be applied. The available procedures allow checks for various structural properties of systems of linear difference equations using an algebraic analysis approach, cf., e.g., [1, 10, 12, 13], for instance, controllability and parametrizability of the behavior.

Both Maple packages `LDA` and `OreModules` are freely available [6], [2].

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