Order Bounds for a Difference Decomposition Algorithm

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Given a system of polynomial difference equations $F = 0, F = f_1, \ldots, f_r$, it is often difficult to determine if the system has a solution, let alone what the solutions are. Difference algebraically, this problem becomes one of determining membership in the perfect difference ideal generated by F. A membership test for the perfect difference ideal generated by a finite collection of difference polynomials can also be used to test for difference algebraic dependencies of other difference polynomials. It is thus essential to have algorithmic methods for determining membership in a perfect difference ideal, and to have complexity estimates for them.

An algorithm to solve this problem was developed in [1]. Given a finite system of difference polynomials over the field of rational functions with automorphism mapping $f(x) \in \mathbb{Q}(x)$ to $f(x+1) \in \mathbb{Q}(x)$, the algorithm ultimately produces what the authors call a Ritt-Wu decomposition of the zero set of the given system. The authors show that this algorithm will either output a useful decomposition of the zero set to solve the system or indicate that the system has no solution. This then, in addition, allows us to test for membership in the perfect difference ideal generated by a finite collection of difference polynomials.

For a complexity bound on this algorithm, we seek a bound on the orders of the difference polynomials produced at each step of the algorithm in terms of the original system. One of the key steps of the algorithm involves the computation of difference ascending chains associated to the given collection of difference polynomials. In this talk, we will present the first known theoretical upper bound for the orders of the ascending chains produced in this algorithm. The bound depends on the number of difference variables, the order and degree of the original system, and the number of elements in the original system. We achieved this bound by proving an effective version of the well-known result by Ritt and Doob which showed that every strictly decreasing sequence of difference ascending chains is finite.

References

[1] X.-S. Gao, Y. Luo, and C. Yuan, *A Characteristic Set Method for Ordinary Difference Polynomial Systems*, Journal of Symbolic Computation 44 (3), 242-260 (2009).